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**SOLUTIONS OF 3D MID-FREQUENCY ACOUSTO-ELASTIC  
SCATTERING ON DISTRIBUTED MEMORY MASSIVELY  
PARALLEL MIMD COMPUTERS**

By

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Final Report

For the period March 25, 1993 through March 30, 1996

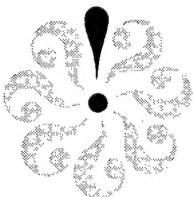
Prepared for  
Naval Research Laboratory  
4555 Overlook Ave., S.E.  
Washington, D.C. 20375-5326

Under

**Research Contract N00014-93-K-2023**

Dr. Susan K. Numrich, Scientific Officer

Submitted by the  
**Old Dominion University Research Foundation**  
**P.O. Box 6369**  
**Norfolk, VA 23508-0369**



September 1996

DTIC QUALITY INSPECTED 1

19960909 132

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
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1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE SEPTEMBER 1996	3. REPORT TYPE AND DATES COVERED FINAL REPORT-1993 MARCH 30-1996 MARCH 30		
4. TITLE AND SUBTITLE SOLUTIONS OF 3D MID-FREQUENCY ACOUSTO-ELASTIC SCATTERING ON DISTRIBUTED MEMORY MASSIVELY PARALLEL COMPUTERS		5. FUNDING NUMBERS C-N00014-93-K-2023		
6. AUTHOR(S) C. E. GROSCH, T. AGARWAL, AND M. ZUBAIR				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) OLD DOMINION UNIVERSITY RESEARCH FOUNDATION P.O. BOX 6369 NORFOLK, VA 23508-0369 DEPARTMENT OF COMPUTER SCIENCE		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) NAVAL RESEARCH LABORATORY 4555 OVERLOOK AVE., S.W. WASHINGTON, D.C. 20375-5326		10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION / AVAILABILITY STATEMENT  UNCLASSIFIED; UNLIMITED		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words)  A general purpose solver for large, sparse, banded systems of equations has been developed and tested on the CM5E. This tool can be applied to any problem arising in finite element analysis such as structures, machinery, acoustic-structure interaction among many others. For a very large problem a factorization and solution takes 10 to 12 minutes; solutions for additional right hand sides will take on the order of one minute each.				
14. SUBJECT TERMS			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT Unclassified	

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## Methodology

The steps in code for solving large, sparse, banded systems of equations are outlined in Table 1. The solver uses either Gaussian Elimination or Cholesky Decomposition. Figure 1 is a schematic flow chart of the solver. A complete solution has three steps: (1) factorization, requiring the updating of data in local node memory, block broadcast of data to other nodes, block receive of data from other nodes and updating of local data using the received data; (2) forward elimination, requiring updating of variables local to the node and broadcasting of update information to other nodes; (3) backward substitution, requiring solving for the variables local to each node and the sending of update information to other nodes. Multiple right hand sides can be used with a single factorization.

Implementation of this algorithm on the CM5E posed several problems. The major problem was the limitations imposed by CM FORTRAN. It is desirable to have the matrix data distributed equally to the nodes in a block round-robin fashion in order to optimize the usage of the four vector pipelines per node. As is sketched out in Figure 2, the CM FORTRAN compiler distributes a vector of  $N$  elements, a row of the matrix in this algorithm, uniformly to the four vector pipelines as  $N/4$ ,  $N/4$ ,  $N/4$  and  $N/4$ . Operations on elements in any one pipeline in effect disables the other pipelines. Thus very low vector performance results. As is sketched in Figure 3, redefining a vector as an array with the second dimension parallel solved this problem. The speedup over the "natural" data distribution was as high as 80.

## Results

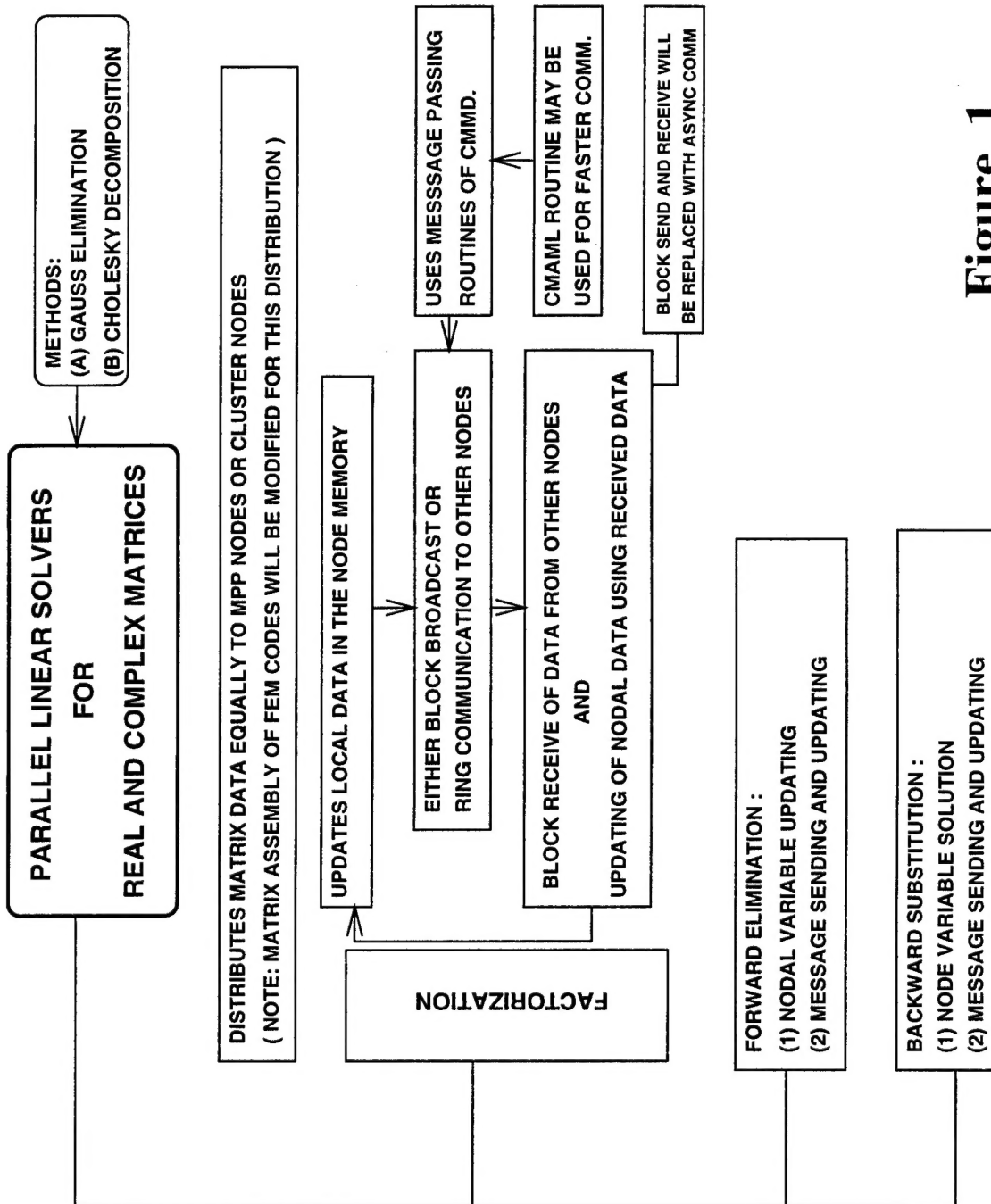
A series of test problems have been run to measure the performance of the code. The problems are: (A): 10,000 equations with an average half band width of 500. (B): 50,000 equations with an average half band width of 1000. (C): 100,000 equations with an average half band width of 4000. The results are presented in the table below. In this table (\*) denotes an estimated value.

Problem	Number of CM5 Nodes	Run Time (Seconds)	Processing Rate (Megaflops)
A	1	21.0	
	4	10.1	
	16	6.5	
	64	5.5	
	128	5.5	
B	1	*1666	*30
	64	76	657
	128	65	760
	256	59	847
C	1	*40000	*40
	64	1047	1538
	128	723	2222
	256	681	2349

Small problems, such as (A), are communication bound and no improvement in performance occurs with more than 4 to 8 nodes. Medium size problems, such as (B) can effectively use up to 64 nodes with modest improvement for additional nodes; again communication cost limits performance to just under a Gigaflop. Large problems, such as (C), can effectively use 128 nodes with modest improvement for 256 nodes; communication dominates for large numbers of nodes but performance of 2.2 to 2.3 Gigaflops is achieved.

## Summary

A general purpose solver for large, sparse, banded systems of equations has been developed and tested on the CM5E. This tool can be applied to any problem arising in finite element analysis such as structures, machinery, acoustic-structure interaction among many others. For a very large problem a factorization and solution takes 10 to 12 minutes; solutions for additional right hand sides will take on the order of one minute each.



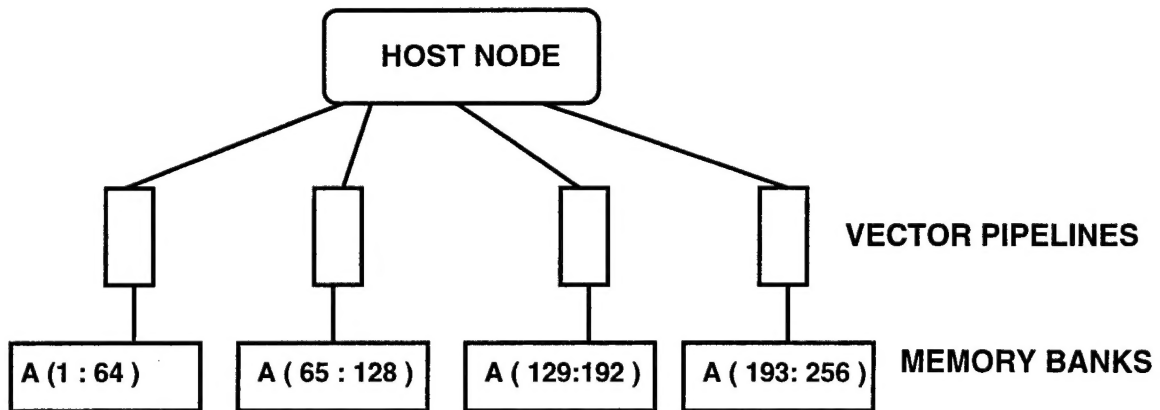
**Figure 1**

## Figure 2

### CMFORTHAN LIMITATIONS

- DISTRIBUTES DATA AMONG FOUR VECTOR PIPELINES AT COMPILE TIME.
- LOOP OPERATION ON SEGMENTS OF THE DECLARED VECTOR TAKE FIXED TIME
- FLOP RATES AS LOW AS 0.5 MFLOPS FOR LONG VECTORS.

EX: PARALLEL ARRAY A (256)



- OPERATIONS ON FIRST 16 ELEMENTS USE ALL VECTOR PIPELINES AND USE SAME ELAPSED TIME AS OPERATING ON ALL 256 ELEMENTS.

## Figure 3

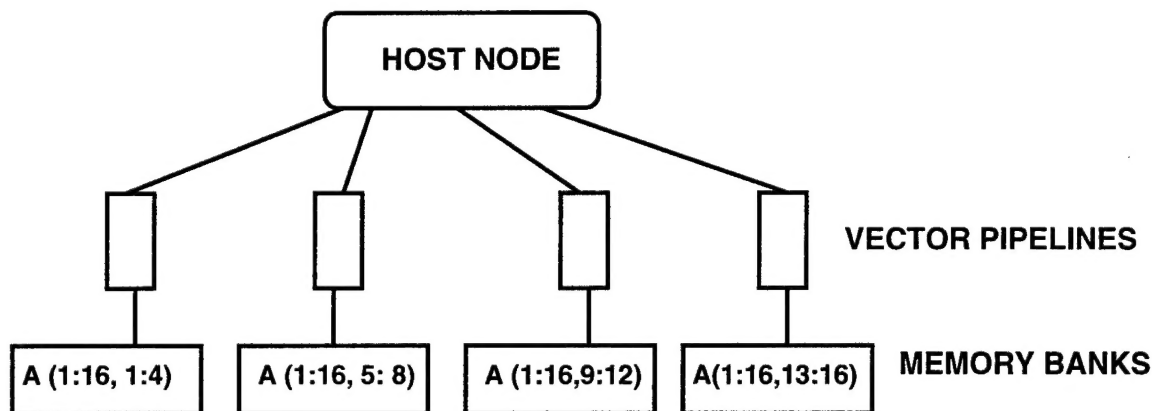
### MATRIX DATA MODIFICATION FOR VECTOR EFFICIENCY UNDER CMFORTRAN

- ARRAY A (256) IS REDEFINED AS TWO DIMESIONAL VECTOR WITH AXIS 2 AS PARALLEL DIMENSION.

CHANGES - ARRAY A (256) ---- ARRAY A (16, 16)

- FLOP RATES AS HIGH AS 40.0 MFLOPS FOR LONG VECTORS.

EX: PARALLEL ARRAY A (16, 16)



- OPERATION ON FIRST 16 ELEMENTS ON SECOND DIMENSION USES ALL FOUR VECTOR PIPELINES.